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Solving the oscillating electric dipole equation by Adomian decomposition technique

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Abstract

For modeling the electric organ discharge of Malapterurus electricus catfish we have set up the electromechanical scheme of the catfish as an electric dipole the charges of which are linked by a spring of stiffness k. We call this particular system of charges an oscillating electric dipole. Taking into account the main forces applied to this dipole and applying the Newton's second law of motion we obtain a relation that we call oscillating electric dipole equation. Our paper reports the resolution of such a nonlinear differential equation. The oscillating electric dipole approach is applied to electric behavior of Malapterurus electricus catfish in order to design the electric equivalent scheme of the fish.

Keywords: Electric charges, oscillating electric dipole equation, electric fish, electric organ discharge, Adomian decomposition technique.

Introduction

Electrostatics remain the branch of physics which studies the phenomena related to fixed electric charges and is a part of Electromagnetism, [1]. As one characterizes the universal gravitation force by associating with the body the mass m which determines its weight, one can characterize the electric state of the body by its electric charge q. There are two kinds of charge. Matter as we ordinarily experience it can be regarded as composed of three kinds of elementary particles, the proton, the neutron and the electron. The electric properties of matter can be explained by its electronic structure. A positive and negative charge of equal magnitude q placed a distance 2a apart constitute an electric dipole, see figure 1. We derive an analytical expression for the electric potential V (M) at any point M of the space due to an electric dipole provided only that the point is not too close to the dipole, [2]. The point M is specified by giving the quantities r and ϴ in figure 1. The potential at any point due to a group of two point charges is found by calculating the potential V_n, n = 1, 2 due to each charge, as if the other charge was not present and added the quantities so obtained:

$$V(M) = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_2}{r_2} - \frac{q_1}{r_1}\right) = \frac{q\,(r_1 - r_2)}{4\pi\varepsilon_0 (r_1 . r_2)}, \quad (1)$$

Where q_1, q_2 are the magnitude of the charges, ($q_1 = q_2 = q$) and r_1 and r_2 stand for the distance from each charge to the point M. We limit consideration to points such that r >> 2a. Then follow these approximate relations from figure 1:

$$r_1 - r_2 \approx 2a\cos\theta \ \text{ and } \ r_1 . r_2 \approx r^2, \quad (2)$$

We need only to find V(r, ϴ) which reduces to:

$$V(M) = \frac{2aq}{4\pi\varepsilon_O r^2} cos\theta \quad (3)$$

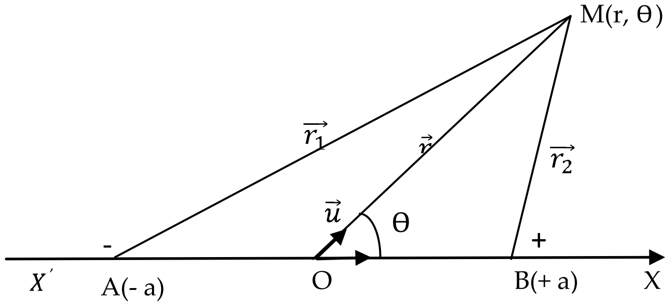


Fig. 1. An electric dipole

When the two charges of an electric dipole are linked by a spring of stiffness k as it is shown in figure 2, we call it the oscillating electric dipole, OED. This study is devoted to solving the equation of the oscillating electric dipole. In section 2, we establish the equation. In section 3 we provide the analytical solution to the oscillating dipole equation, [3], [4]. In section 4 the electric potential we have obtained by means of that analytical solution is applied for modeling the electric behavior of Malapterurus electricus catfish. In section 5 follows the conclusion.

2. MATHEMATICAL MODEL

We consider two equal electric charges q of opposite sign possessing equal masse m. The forces acting on these charges are in order: the gravitational force F_G, the force of viscous friction F_V, the repelling force of the spring F_R and the generalized Lorentz force F_L. The analytical expression of each force is as following:

$$F_G = -G\frac{m_1.m_2}{d^2},\ F_V = -\rho v,\ F_R = -kx\ , \qquad (4)$$

$$F_L = F_C + F_{compl}, \qquad (5)$$

$$F_C = -\frac{1}{4\pi\varepsilon_0}\frac{q_1.q_2}{d^2},\ F_{compl} = q.\left|\vec{v}x\vec{B}\right|, \qquad (6)$$

Where d is the distance between the center of gross masses m_1 and m_2 mainly resulting masses of protons and neutrons contained in the nucleus of every atoms of the considered charges q_1 and q_2, $G = 6.67\ 10^{-11}$ N.m²/kg², [5]; ρ the coefficient of viscous friction, v the linear velocity; k the stiffness of the spring and x the elongation; F_C stands for the Coulomb force and $\varepsilon_0 \approx 8.85.10^{-12}$ Farad/m, [5]. Each moving charge will create a magnetic field B as it is shown by Batailler [6] throughout Rowland's experience. When an electric charge q moves with the speed v in a magnetic field it experiences an additional force F_{compl} on top of Coulomb one. In what follows, we neglect the component F_{compl} as we adopt the assumption that the magnetic field is negligible as v/c is small [2]. According to Newton's second law, the equation of motion of the oscillating electric dipole can be written as:

$$m\frac{d^2x}{dt^2} = F_G + F_{v+}F_{R+}F_C, \qquad (7)$$

For the oscillating electric dipole, $d = 2(x+a)$, where $2a$ is the distance between the charges at the equilibrium and $v = \frac{dx}{dt}$. It is well known that the gravitational force is about 10^{31} lower than Coulomb one and can be neglected. The equation (7) is equivalent to:

$$\frac{d^2x}{dt^2} + \alpha\frac{dx}{dt} + \beta x + \frac{\gamma}{[2(x+a)]^2} = 0, \qquad (8)$$

Where,

$$\alpha = \frac{\rho}{m};\quad \beta = \frac{k}{m},\ \gamma = \frac{q^2}{4\pi m\varepsilon_0}, \qquad (9)$$

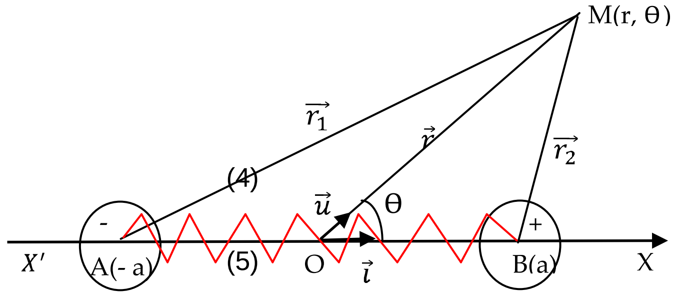


Fig. 2. An oscillating electric

2.1 Definition *The equation (8) is called the oscillating electric dipole equation*

We impose the initial conditions to be:

$$x(t_0) = u,\ \dot{x}(t_0) = 0, \qquad (10)$$

3. Analytical solution

In this section we provide the analytical solution to (8) where x is the unknown function depending on the time, taking into account (10). Then the equation (8) is equivalent to the non linear integral equation, [7]:

$$x(t) - \int_{t_0}^{t} \frac{g(t)h(s)-g(s)h(t)}{W(g(s),h(s))} \frac{\gamma}{[2(x(t)+a)]^2} ds = bg(t) + ch(t), \quad (11)$$

Where:

$$W(g(s), h(s)) = \begin{vmatrix} g(s) & h(s) \\ g'(s) & h'(s) \end{vmatrix}; \; b = \frac{x(t_0)h'(t_0)}{W(g(t_0),h(t_0))}; = \frac{-x(t_0)g'(t_0)}{W(g(t_0),h(t_0))}, \quad (12)$$

And g, h are the particular solutions of homogeneous equation related to the equation (8)

$$g(t) = e^{-0.5\alpha t}.cos\left(0.5\sqrt{4\beta-\alpha^2}t\right), \quad h(t) = e^{-0.5\alpha t}.sin\left(0.5\sqrt{4\beta-\alpha^2}t\right), \text{ for } \alpha^2 < 4\beta, \quad (13)$$

Thus we can write the non linear functional equation (11) in the form:

$$x - G(x) = p, \quad (14)$$

Where,

$$G(x) = \int_{t_0}^{t} \frac{g(t)h(s)-g(s)h(t)}{W(g(s),h(s))} \frac{\gamma}{[2(x(s)+a)]^2} ds, \quad p(t) = bg(t) + ch(t), \quad (15)$$

G is a nonlinear operator from a Hilbert space H into H; *p* is a given function in H. We assume that (14) has a unique solution. The Adomian technique [8] defines the solution of (14) by the decomposition series $x = \sum_{n\geq 0} x_n$ using the following scheme:

$$x_0 = p, \quad (16)$$

$$x_{n+1} = A_n = \frac{1}{n!}\left[\frac{d^n}{\lambda^n} G\left(\sum_{i=0}^{+\infty} \lambda^i x_i\right)\right]_{\lambda=0}; \quad n \geq 0, (17)$$

The 2-term approximation of $x = \sum_{n\geq 0} x_n$ can be written in the form, for $\alpha^2 < 4\beta$

$$x(t) = be^{-0.5\alpha t}.cos\left(0.5\sqrt{4\beta-\alpha^2}\right)t + ce^{-0.5\alpha t}.sin\left(0.5\sqrt{4\beta-\alpha^2}\right)t +$$

$$\frac{2\gamma}{\sqrt{4\beta-\alpha^2}} \int_{t_0}^{t} \frac{e^{-0.5\alpha(t-s)}.sin\left(0.5\sqrt{4\beta-\alpha^2}\,(s-t)\right)}{\left\{e^{-0.5\alpha s}\left(b.cos\,0.5\sqrt{4\beta-\alpha^2}s\right)+c.sin\left(0.5\sqrt{4\beta-\alpha^2}\,s\right)+2\alpha\right\}} ds, \quad (18)$$

(11)

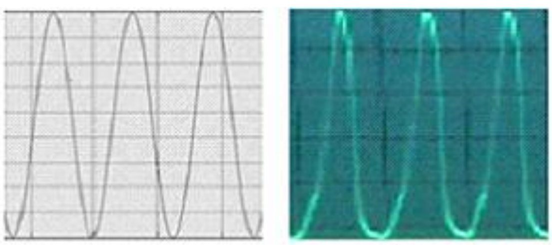
Therefore the electric potential of the OED is as following:

$$V = \frac{2q(a+x(t))}{4\pi\varepsilon_0 r^2}\cos\theta, \quad (19)$$

4. Applying the solution of the Equation in Malapterurus electricus EOD modeling (12)

We have investigated the electric behavior of Malapterurus electricus to test the oscillating electric dipole that we consider as the equivalent electromechanical model of the catfish. The model leads to the non linear differential (14) equation of second order (8). As we have characterized the electric shocks of Malapterurus electricus catfish in our Laboratory [9] some data of the Electric Organ Discharge, EOD of the catfish are available: these EOD are similar to mono or two fold alternating (15) electric waves frequency and magnitude of which vary depending on the fish. Taking into account (19) we realize a computer simulation in Matlab. The results of the simulation and the EOD of the catfish are very close each to other as we can see in figure 3 and seem like the output voltage of a Graetz bridge rectifier. Therefore the design of Malapterurus electricus catfish's EOD simulator is conceivable.

(16)



a b

Fig. 3. a- Computer simulation of the model

b- Electric organ discharge of the catfish

5. Conclusion

The EOD of Catfish in general and Malapterurus electricus 'one in particular are powerful and can supply some applications in energy. Malapterurus electricus EOD overcome sometime 350 Volts [10] and the catfish is able to give so many EOD per second, [9]: it is a true source of energy worthy of interest. In order for Malapterurus electricus catfish energy supplying to be we must set up the equivalent electric scheme of the fish. For that purpose we have solved the non linear second order differential equation given by the electromechanical model that we got for the fish. Each parameter α, β and γ of (8) plays a role: when α increases the magnitude of the electric shocks of the model decreases and when β increases the frequency of these shocks increases. The magnitude of the shocks is proportional to γ. The electric equivalent scheme we got for the fish allows us to design a mosquitoes 'larvae killer in fresh water to control malaria disease burden. Elsewhere we succeed with the help of the scheme in the design of a 12 Volts accumulator battery charger with the energy of the catfish. We hope to use the electric shocks of the model to cure some sickness as Egyptian did formerly with the help of Malapterurus electricus catfish.

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